MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

TMA1101 – CALCULUS

(All sections / Groups)

13 FEBRUARY 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of five pages with **FIVE** questions.

2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the answer booklet provided.

4. **You are required to write proper steps. No calculators are allowed.**

Question 1 (10 marks)

(a) Find the following limit.

[*You must show at least one intermediate step where* $\lim_{x\to c}$ *is still needed.*]

(i) $\lim_{x\to 5}\left(\dfrac{x^2-25}{x-5}\right)$

(ii) $\lim_{x\to\infty}\left(\dfrac{x^2+3}{8x^3+9x+1}\right)$

[2.5 marks]

(b) Given $f(x)=\begin{cases}1+x^3, & x<0\\ x^2-1, & x\geq 0\end{cases}$

(i) Determine $\lim_{x\to 0^-} f(x)$ and $\lim_{x\to 0^+} f(x)$.

[*For this part, you must show at least one intermediate step where* $\lim_{x\to 0^-}$ *or* $\lim_{x\to 0^+}$ *is still needed.*]

(ii) Does $\lim_{x\to 0} f(x)$ exist? Give your reason. If it exists, state its value.

(iii) Is the function $f(x)$ continuous at $x=0$? Give your reason for your answer.

[4 marks]

(c) (i) State the **intermediate value theorem** (i.e., the full statement including the hypothesis and the conclusion).

(ii) Show that there is a root of the equation $x^2-x-12=0$ between 3 and 5.

You must write proper steps to arrive at conclusion; just writing some calculations would not be enough.

[3.5 marks]

Continued.....

Question 2 (10 marks)

(a) Use the formal definition of the derivative to compute $f'(-1)$ when $f(x) = x^2 + 1$. ***You are reminded to write proper steps.***

[2.5 marks]

(b) Find $\frac{dy}{dx}$ with y as given.

[***Use the product rule or the quotient rule for differentiation; show proper steps.***]

(i) $y = x^3 \cos(2x)$

(ii) $y = \frac{e^{2x}}{\ln x}$

[3 marks]

(c) The point (5, 3) lies on the curve $xy + y - 4x = 2$.

Use **implicit differentiation** to obtain $\frac{dy}{dx}$ **in terms of x and y.**

Then determine the gradient of the tangent to the curve $xy + y - 4x = 2$ at the point (5, 3).

[4.5 marks]

Continued.....

Question 3 (10 marks)

(a) Solve the following integral using **integration by substitution** with $u = x^2 + 1$.

$$\int x\cos(x^2+1)\,dx$$

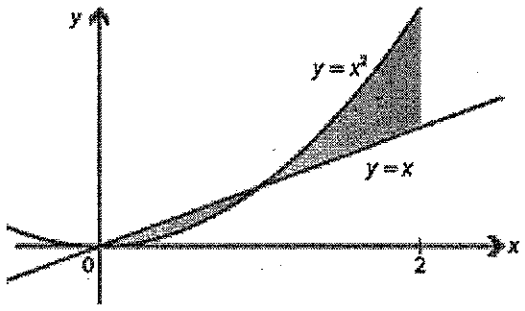
[2 marks]

(b) (i) Use $\cos\theta = \dfrac{e^{i\theta} + e^{-i\theta}}{2}$ and $\sin\theta = \dfrac{e^{i\theta} - e^{-i\theta}}{2i}$ to find the values of A and B which make the equation $\sin 5\theta\cos 3\theta = \dfrac{1}{2}\left(\sin A\theta + \sin B\theta\right)$ an identity.

(ii) Then use the result obtained from (i) to evaluate $\int_0^{\frac{\pi}{8}} \sin 5\theta\cos 3\theta d\theta$.

[4 marks]

(c) The following figure shows two regions bounded by parabola $y = x^2$, straight line $y = x$ and vertical line $x = 2$.



(i) Determine the x-coordinates of the points of intersection between the parabola and the straight line.

(ii) Find the total area of the bounded region.

[4 marks]

Continued.....

Question 4 (10 marks)

(a) Given a sequence $\{a_n\}$ with $a_n = \dfrac{3n^3 - 2n}{5n^3 + 2}$.

(i) Determine $\lim_{n\to\infty} a_n$. You are reminded to write proper steps.

(ii) Then determine whether the infinite series $\sum_{n=1}^{\infty} \dfrac{3n^3 - 2n}{5n^3 + 2}$ is convergent. Give the reason for your answer.

[2.5 marks]

(b) Determine whether the geometric series $\sum_{n=0}^{\infty} \left(-\dfrac{1}{2}\right)^x$ is convergent. If the series is convergent, find its sum.

[2 marks]

(c) Determine if the infinite series $\sum_{n=1}^{\infty} \dfrac{4\sqrt{n}}{n^4}$ converges by using the *p*-series test.

[1.5 marks]

(d) Find the Maclaurin polynomial of order 3 for $f(x) = x^3 + e^{2x}$.

[4 marks]

Continued.....

Question 5 (10 marks)

(a) Given $F(x,y) = y^2\cos(x) + \sin(y) - 5x$, find the **partial derivatives** $\frac{\partial F}{\partial x}$ and $\frac{\partial F}{\partial y}$.

[1 mark]

(b) Solve the first order **separable equation** $\frac{dy}{dx} = \frac{4\sin x}{3y^2}$ subject to the initial condition $y(0) = 2$. You may leave your answer in implicit form.

[2 marks]

(c) You are told that e^{-x^2} is an integrating factor for the first order linear equation $\frac{dy}{dx} - 2xy = x$. Solve the equation and give your solution in explicit form.

[3 marks]

(c) Consider the second order differential equation

$$y'' - 2y' - 3y = 2 + x$$

(i) Find the roots of the **characteristic equation** of the corresponding homogeneous differential equation. Then write down the general solution y_h of this homogeneous differential equation.

(ii) If $y = Ax + B$ is a **particular solution** of the differential equation $y'' - 2y' - 3y = 2 + x$. Determine the values of A and B.

(iii) Hence, write down the **general solution** for the differential equation $y'' - 2y' - 3y = 2 + x$.

[4 marks]

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